

100/06/15微積分二練習題 學號_____姓名_____

(A) 1. 求定積分值 $\int_{-1}^1 (x^5 + x) dx =$ (A) 0 (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) 以上皆非

(C) 2. 求定積分值 $\int_0^1 4x^3 dx =$ (A) 0 (B) 4 (C) 1 (D) 以上皆非

(B) 3. 求定積分值 $\int_0^1 x^2 dx =$ (A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{9}$ (D) 以上皆非

(C) 4. 求定積分值 $\int_0^1 \int_0^1 (3x^2 + 3y^2) dx dy =$ (A) 0 (B) 1 (C) 2 (D) 以上皆非

(A) 5. 設 $f(x, y) = x^6 y^7$ ，則 $\frac{\partial f}{\partial x} =$ (A) $6x^5 y^7$ (B) $7x^6 y^6$ (C) $6x^5 + 7y^6$ (D) 以上皆非

1. 設 $f(x, y) = 3x^4 - 2x^3 y^4 + 4y^7 - 9x + 8y + 7$ ，

$$\frac{\partial f}{\partial x} = 12x^3 - 6x^2 y^4 - 9 \quad \frac{\partial f}{\partial y} = -8x^2 y^3 + 28y^6 + 8$$

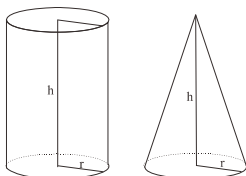
$$\frac{\partial^2 f}{\partial x^2} = 36x^2 - 12xy^4 \quad \frac{\partial^2 f}{\partial y^2} = -24x^2 y^2 + 168y^5$$

2. 求疊積分值 $\int_0^1 \int_0^2 6x^2 y dx dy =$ 8

$$\int_0^1 \int_0^2 6x^2 y dx dy = \int_0^1 2x^3 y \Big|_{x=0}^{x=2} dy = \int_0^1 16y dy = 8y^2 \Big|_0^1 = 8$$

3. 求疊積分值 $\int_0^1 \int_1^2 (2xy + 2y) dy dx =$ 9/2

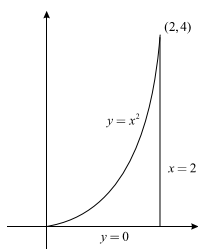
4. 試證明圓柱之體積為圓錐體積之 3 倍



$$dV = \pi y^2 dx = \pi \frac{r^2}{h^2} x^2 dx$$

$$V = \int_0^h \pi \frac{r^2}{h^2} x^2 dx = \pi \frac{r^2}{h^2} \frac{1}{3} x^3 \Big|_0^h = \frac{1}{3} \pi r^2 h$$

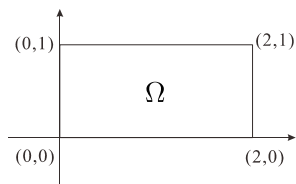
5. 試求 $y = x^2$ ， $x = 2$ ， $y = 0$ 所圍區域之面積



$$dA = ydx = x^2 dx$$

$$A = \int_0^2 x^2 dx = \frac{1}{3} x^3 \Big|_0^2 = \frac{8}{3}$$

6. 3. 設 $f(x, y) = 6xy^2$, Ω 如下圖 , 求 $\iint_{\Omega} f(x, y) dA$



$$\iint_{\Omega} f(x, y) dA = \int_0^1 \int_0^2 6xy^2 dx dy$$

$$= \int_0^1 3x^2 y^2 \Big|_{x=0}^{x=2} dy = \int_0^1 12y^2 dy = 4y^3 \Big|_0^1 = 4$$