微積分二　第 10 週：定積分、面積與旋轉體體積複習

1. $\int_0^2 x^2 dx =$ $\frac{8}{3}$

$$\int_0^2 x^2 dx = \frac{1}{3}x^3\Big|_0^2 = \frac{8}{3}$$

2. $\int_1^4 (x+2)\,dx =$ $\frac{27}{2}$

$$\int_1^4 (x+2)\,dx = \frac{1}{2}x^2 + 2x\Big|_1^4$$
$$= \left(\frac{16}{2}+8\right) - \left(\frac{1}{2}+2\right) = \frac{15}{2}+6 = \frac{27}{2}$$

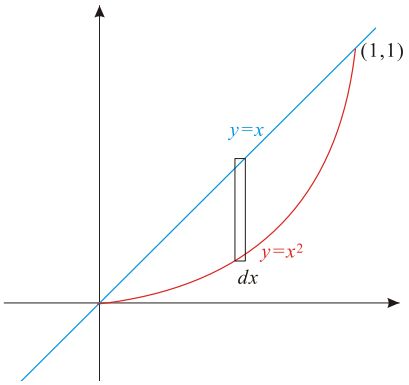
3. $\int_0^4 \sqrt{x}\,dx =$ $\frac{16}{3}$

$$\int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^4 = \frac{2}{3}\cdot 8 = \frac{16}{3}$$

4. $\int_0^3 (x^2+2)\,dx =$ 15

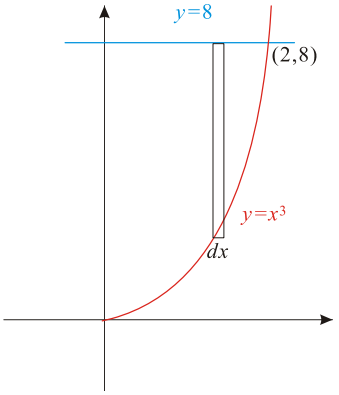
$$\int_0^3 (x^2+2)\,dx = \frac{1}{3}x^3 + 2x\Big|_0^3 = 9+6 = 15$$

5.求 $y=x$, $y=x^2$ 所圍區域之面積



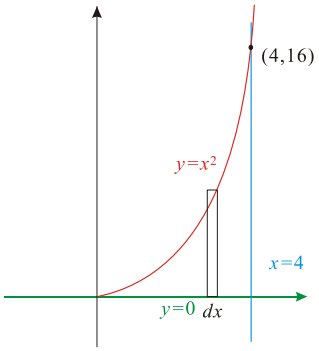
$$dA = (x-x^2)dx$$
$$A = \int_0^1 (x-x^2)dx = \frac{1}{6}$$

6.求 $y=x^3$, $y=8$, $x=0$ 所圍區域之面積



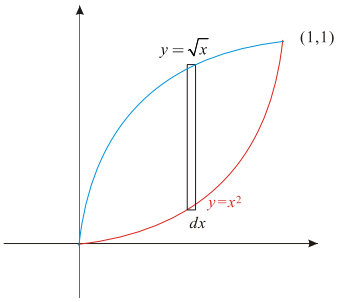
$$dA = (8-x^3)dx$$
$$A = \int_0^2 (8-x^3)dx = 12$$

7.求 $y=x^2$, $x=4$, $y=0$ 所圍區域之面積



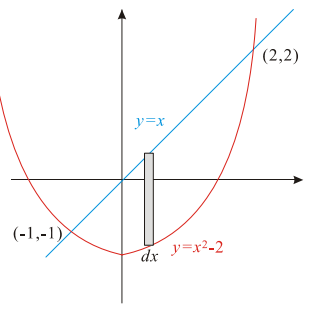
$$dA = (x^2-0)dx$$
$$A = \int_0^4 x^2 dx = \frac{64}{3}$$

8.求 $y=x^2$, $y=\sqrt{x}$ 所圍區域之面積



$$dA = (\sqrt{x}-x^2)dx$$
$$A = \int_0^1 (\sqrt{x}-x^2)dx = \frac{1}{3}$$

9. $y=x^2-2$, $y=x$ 所圍區域之面積



$$\begin{cases} y=x^2-2\cdots\cdots(1) \\ y=x\cdots\cdots(2) \end{cases}$$

$x^2-x-2=0 \quad \Rightarrow (x+1)(x-2)=0$

$x=-1 \quad , \quad x=2$

$dA=[x-(x^2-2)]dx$

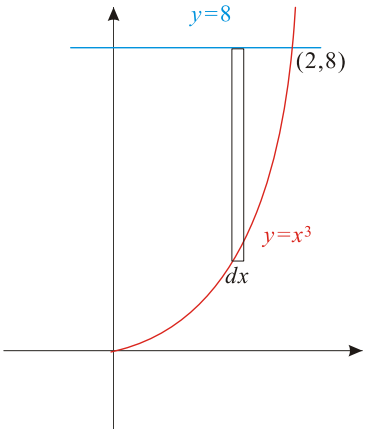
$$A=\int_{-1}^{2}[x-(x^2-2)]dx=\int_{-1}^{2}(x-x^2+2)dx$$

$$=\frac{1}{2}x^2-\frac{1}{3}x^3+2x\Big|_{-1}^{2}=\left(\frac{4}{2}-\frac{8}{3}+4\right)-\left(\frac{1}{2}+\frac{1}{3}-2\right)$$

$$=\frac{3}{2}-\frac{9}{3}+6=\frac{9}{2}$$

10.求 $y=x^3$, $y=8$, $x=0$ 所圍區域繞(1) x 軸 (2) y 軸旋轉之體積

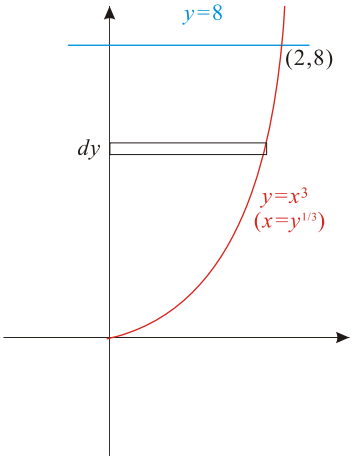
(1)繞 x 軸(圓盤法)



$dV=\left[\pi 8^2-\pi(x^3)^2\right]dx$

$$V=\pi\int_0^2\left[64-x^6\right]dx=\pi\left(64x-\frac{1}{7}x^7\right)\Big|_0^2=\frac{768\pi}{7}$$

(2) y 軸(圓盤法)

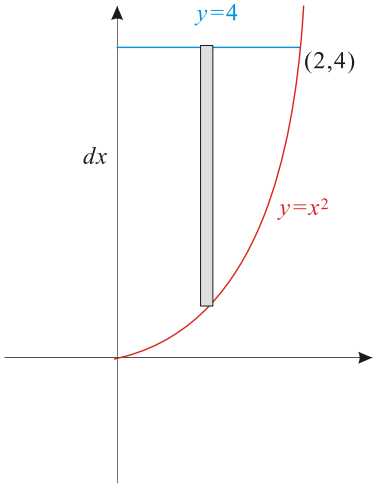


$dV=\pi x^2dy=\pi y^{\frac{2}{3}}dy$

$$V=\int_0^8\pi y^{\frac{2}{3}}dy=\pi\frac{3}{5}y^{\frac{5}{3}}\Big|_0^8=\pi\frac{3}{5}\cdot 32=\frac{96\pi}{5}$$

11. 求 $y=x^2$，$y=4$，$x=0$ 在第一象限所圍區域繞(1) x 軸(2) y 軸旋轉之體積

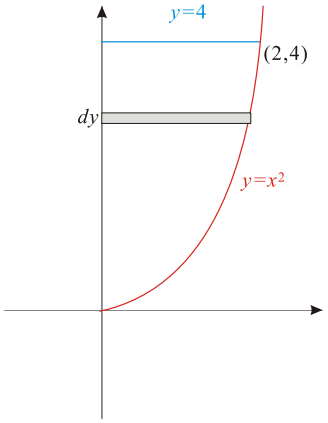
(1) 繞 x 軸(圓盤法)



$$dV=\left[\pi 4^2-\pi(x^2)^2\right]dx$$

$$V=\pi\int_0^2\left[16-x^4\right]dx=\pi\left(16x-\frac{1}{5}x^5\right)\Big|_0^2=\frac{128\pi}{5}$$

(2)繞 y 軸(圓盤法)

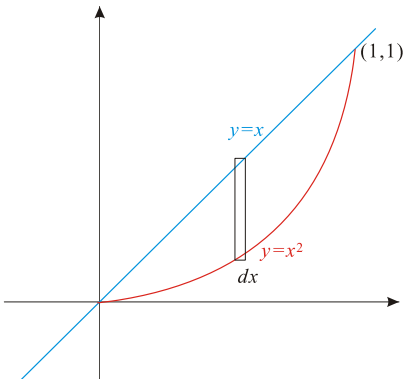


$$dV=\pi x^2dy=\pi(y^{\frac{1}{2}})^2dy$$

$$V=\int_0^4\pi ydy=\pi\frac{1}{2}y^2\Big|_0^4=\pi\cdot 8=8\pi$$

12. 求 $y=x$，$y=x^2$ 所圍區域繞(1) x 軸(2) y 軸旋轉之體積

(1)繞 x 軸(圓盤法)

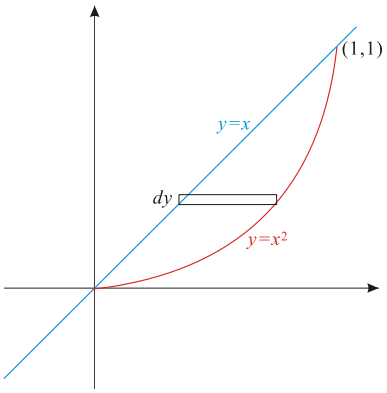


$$dV=\left[\pi x^2-\pi(x^2)^2\right]dx$$

$$V=\pi\int_0^1\left[\pi x^2-\pi(x^2)^2\right]dx$$

$$=\pi\left(\frac{1}{3}x^3-\frac{1}{5}x^5\right)\Big|_0^1=\frac{2\pi}{15}$$

(2)繞 y 軸(圓盤法)



$$dV=\left[\pi(y^{\frac{1}{2}})^2-\pi(y)^2\right]dy$$

$$V=\pi\int_0^1\left[\pi y-\pi y^2\right]dy$$

$$=\pi\left(\frac{1}{2}y^2-\frac{1}{3}y^3\right)\Big|_0^1=\frac{\pi}{6}$$