

假設檢定例題(雙母體、獨立小樣本)

(1) 當 $\sigma_1 = \sigma_2$ 時

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{where } s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \quad \text{自由度} = n_1 + n_2 - 2$$

(2) 當 $\sigma_1 \neq \sigma_2$ 時

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{自由度} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

1. 某航空公司爲了加強空服人員禮儀，特進行爲期三天的訓練，位了解其成效，隨機訪問了 12 位旅客做評分

無訓練	有訓練
$\bar{x}_1 = 74$	$\bar{x}_2 = 84$
$s_1 = 7$	$s_2 = 4$
$n_1 = 12$	$n_2 = 12$

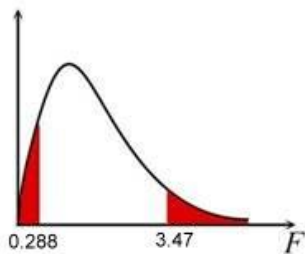
試以 $\alpha = 0.05$ 檢定 $H_0 : \mu_1 = \mu_2$

Sol

(1) 先使用 F 檢定

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$



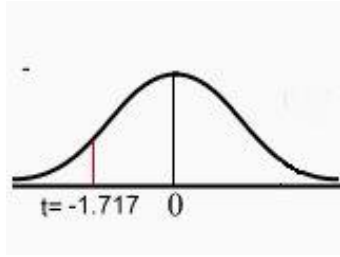
$$F = \frac{s_1^2}{s_2^2} = \frac{49}{16} = 3.06$$

我們接受 $\sigma_1 = \sigma_2$ 的假設

(2) 再使用 t 檢定

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 < \mu_2$$



$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(74 - 84)}{\sqrt{32.5} \sqrt{\frac{1}{12} + \frac{1}{12}}} = -4.297$$

拒絕 H_0

結論：訓練有使旅客滿意度增加

2. 一個雙盲研究

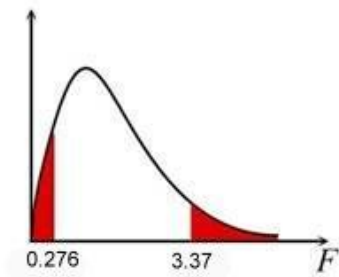
新藥	安慰劑
$\bar{x}_1 = 15$	$\bar{x}_2 = 10.5$
$s_1 = 8$	$s_2 = 4$
$n_1 = 11$	$n_2 = 13$

試以 $\alpha = 0.05$ 檢定新藥的藥效

(1) 先使用 F 檢定

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$



$$F_{0.025(10,12)} = 3.37$$

$$F_{0.975(10,12)} = \frac{1}{F_{0.025(12,10)}} = \frac{1}{3.62} = 0.276$$

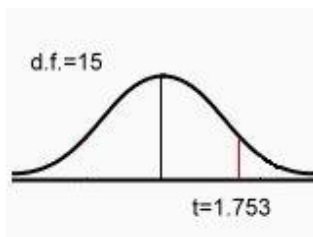
$$F = \frac{s_1^2}{s_2^2} = \frac{64}{16} = 4$$

我們認為 $\sigma_1 \neq \sigma_2$

(2) 再使用 t 檢定

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 > \mu_2$$



$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(15 - 10.5)}{\sqrt{\frac{64}{11} + \frac{16}{13}}} = \frac{4.5}{2.645} = 1.69$$

$$\text{自由度} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \doteq 15.3$$

1.69 < 1.753 故接受 H_0 ，也就是新藥藥效不顯著